

Bogoliubov dispersion relation for a “photon fluid”: Is this a superfluid?*

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We discuss the possibility that photons, which are bosons, can form a 2D superfluid due to Bose-Einstein condensation inside a nonlinear Fabry-Perot cavity filled with atoms in their ground states. A “photon fluid” forms inside the cavity as a result of multiple photon-photon collisions mediated by the atoms during a cavity ring-down time. The effective mass and chemical potential for a photon inside this fluid are nonvanishing. This implies the existence of a Bogoliubov dispersion relation for the low-lying elementary excitations of the photon fluid, and in particular, that sound waves exist for long-wavelength, low-frequency disturbances of this fluid. Possible experiments to test for the superfluidity of the photon fluid based on the Landau critical-velocity criterion will be discussed.

I. INTRODUCTION

Inspired by the recent experimental discoveries of Bose-Einstein condensation of laser-cooled atoms [1–4], we would like to consider here the inverse question: Can one observe Bose-Einstein condensation of *photons*? Closely related is a second question: Is the resulting Bose condensate a superfluid? We know that photons are bosons, so that it would seem that they could in principle undergo this kind of condensation. The difficulty is that in the usual Planck blackbody configuration, which consists of an empty 3D cavity, the photon is massless and its chemical potential is zero, so that the Bose-Einstein condensation of photons under these circumstances would seem to be impossible.

However, we consider here an atom-filled 2D Fabry-Perot cavity configuration, instead of the usual empty 3D Planck cavity configuration. We find that if one excites only one of the longitudinal modes of the Fabry-Perot cavity by means of narrow-linewidth laser radiation, so that the dynamics of the light inside the cavity becomes effectively two-dimensional [5], and if this radiation is well detuned to the red side of the resonance of the atoms in their ground state, so that an effective repulsive photon-photon interaction mediated by the atoms results, the resulting effective mass and chemical potential of a photon inside the cavity no longer vanishes. Thus Bose-Einstein condensation of photons inside the Fabry-Perot cavity *can* occur. We shall explore the circumstances under which this may happen, and shall connect this problem with an earlier problem solved by Bogoliubov for the weakly-interacting Bose gas. In this way, we shall see that the Bogoliubov dispersion relation should hold for the “photon fluid” that forms as a result of multiple photon-photon collisions occurring inside the cavity. In particular, we shall see that sound waves, or “phonons” in the photon fluid, are the lowest-lying excitations of the system. According to an argument due to Landau, this implies that the photon fluid could become a superfluid.

Historically speaking, in the study of the interaction of light with matter, most of the recent emphasis has been on exploring new states of matter, such as the recently observed atomic Bose-Einstein condensates. However, not as much attention has been focused on exploring new states of light. Of course, the invention of the laser led to the discovery of a new state of light, namely the coherent state, which is a very robust one. Two decades ago, squeezed states were discovered, but these states are not as robust as the coherent state, since they are easily degraded by scattering and absorption. In contrast to the laser, which involves a population-inverted atomic system that is far away from thermal equilibrium, we shall explore here states very close to the ground state of a photonic system, and hence very near absolute zero in temperature. Hence they should be robust ones.

The interacting many-photon system (the “photon fluid”) is an example of a quantum many-body problem. In an extension of the interacting Bose gas problem, we shall derive the Bogoliubov dispersion relation for the weakly-

*It is a great pleasure to dedicate this paper to my lifelong friend, Marlan Scully. Marlan and I first met when we were graduate students (of Willis Lamb and Charles Townes, respectively). Over many years we have enjoyed discussing and learning physics together, as well as sharing and growing in our like precious faith. Happy birthday, Marlan!

interacting photon gas with repulsive photon-photon interactions, starting from the microscopic (i.e., the second-quantized) level. Thereby we shall find an expression for the effective chemical potential of a photon in the photon fluid, and shall relate the velocity of sound in the photon fluid to this nonvanishing chemical potential. In this way, we shall lay the theoretical foundations for an experiment to measure the sound wave part of the dispersion relation for the photon fluid. We shall also propose another experiment to measure the critical velocity of this fluid, and thus to test for the possibility of the superfluidity of the resulting state of the light.

Although the interaction Hamiltonian used in this paper is equivalent to that used earlier in four-wave squeezing, we emphasize here the *collective* aspects of the problem which result from multiple photon-photon collisions. This leads to the idea of the “photon fluid.” It turns out that microscopic and macroscopic analyses yield exactly the same Bogoliubov dispersion relation for the excitations of this fluid [6]. Hence it may be argued that there is nothing fundamentally new in the microscopic analysis given below which is not already contained in the macroscopic, classical nonlinear optical analysis. However, it is the microscopic analysis which leads to the viewpoint of the interacting photon system as a “photon fluid,” a conception which could give rise to new ways of understanding and discovering nonlinear optical phenomena. Furthermore, the interesting question of the quantum optical state of the light inside the cavity resulting from multiple collisions between the photons (i.e., whether it results in a coherent, squeezed, Fock, or some other quantum state), cannot be answered by the macroscopic, classical nonlinear optical analysis, and this necessitates the microscopic treatment given below.

II. THE BOGOLIUBOV PROBLEM

Here we re-examine one particular many-body problem, the one first solved by Bogoliubov [7,8]. Suppose that one has a zero-temperature system of bosons which are interacting with each other repulsively, for example, a dilute system of small, bosonic hard spheres. Such a model was intended to describe superfluid helium, but in fact it did not work well there, since the interactions between atoms in superfluid helium were too strong for the theory to be valid. In order to make the problem tractable theoretically, let us assume that these interactions are weak. In the case of light, the interactions between the photons are in fact always weak, so that this assumption is a good one. However, these interactions are nonvanishing, as demonstrated by the fact that photon-photon collisions mediated by atoms excited near, but off, resonance have been experimentally observed [9]. We start with the Bogoliubov Hamiltonian

$$\begin{aligned} H &= H_{free} + H_{int} \\ H_{free} &= \sum_p \epsilon(p) a_p^\dagger a_p \\ H_{int} &= \frac{1}{2} \sum_{\kappa p q} V(\kappa) a_{p+\kappa}^\dagger a_{q-\kappa}^\dagger a_p a_q, \end{aligned} \quad (1)$$

where the operators a_p^\dagger and a_p are creation and annihilation operators, respectively, for bosons with momentum p , which satisfy the Bose commutation relations

$$[a_p, a_q^\dagger] = \delta_{pq} \text{ and } [a_p, a_q] = [a_p^\dagger, a_q^\dagger] = 0. \quad (2)$$

The first term H_{free} in the Hamiltonian represents the energy of the free boson system, and the second term H_{int} represents the energy of the interactions between the bosons arising from the potential energy $V(\kappa)$, which is the Fourier transform of the potential energy $V(r_2 - r_1)$ in configuration space of a pair of bosons located at r_2 and r_1 . The interaction term is equivalent to the one responsible for producing squeezed states of light via four-wave mixing [12]. It represents the annihilation of two particles, here photons, of momenta p and q , along with the creation of two particles with momenta $p + \kappa$ and $q - \kappa$, in other words, a scattering process with a momentum transfer κ between a pair of particles with initial momenta p and q , along with the assignment of an energy $V(\kappa)$ to this scattering process. Here the assumption that the interactions are weak means that the second term in the Hamiltonian is much smaller than the first, i.e., $|V(\kappa)| \ll |\epsilon(\kappa)|$.

III. THE FREE-PHOTON DISPERSION RELATION INSIDE A FABRY-PEROT RESONATOR

Photons with momenta p and q also obey the above commutations relations, so that the Bogoliubov theory should in principle also apply to the weakly-interacting photon gas. The factor $\epsilon(p)$ represents the energy as a function of the momentum (the dispersion relation) for the free, i.e., noninteracting, bosons. In the case of photons in a Fabry-Perot

resonator, the boundary conditions of the mirrors cause the $\epsilon(p)$ of a photon trapped inside the resonator to become an energy-momentum relation which is identical to that of a nonrelativistic particle with an effective mass [9,10] of $m = \hbar\omega/c^2$. This can be understood starting from Fig. 1.

For high-reflectivity mirrors, the vanishing of the electric field at the reflecting surfaces of the mirrors imposes a quantization condition on the allowed values of the z -component of the photon wave vector, $k_z = n\pi/L$, where n is an integer, and L is the distance between the mirrors. Thus the usual frequency-wavevector relation

$$\omega(k) = c[k_x^2 + k_y^2 + k_z^2]^{1/2}, \quad (3)$$

upon multiplication by \hbar , becomes the energy-momentum relation for the photon

$$E(p) = c[p_x^2 + p_y^2 + p_z^2]^{1/2} = c[p_x^2 + p_y^2 + \hbar^2 n^2 \pi^2 / L^2]^{1/2} = c[p_x^2 + p_y^2 + m^2 c^2]^{1/2}, \quad (4)$$

where $m = \hbar n\pi/Lc$ is the effective mass of the photon. In the limit of small-angle (or paraxial) propagation, where the small transverse momentum of the photon satisfies the inequality

$$p_\perp = [p_x^2 + p_y^2]^{1/2} \ll p_z = \hbar k_z = \hbar n\pi/L, \quad (5)$$

we obtain from a Taylor expansion of the relativistic relation, a nonrelativistic energy-momentum relation for the 2D noninteracting photons inside the Fabry-Perot resonator

$$E(p_\perp) \cong mc^2 + p_\perp^2/2m, \quad (6)$$

where $m = \hbar n\pi/Lc \cong \hbar\omega/c^2$ is the effective mass of the confined photons. It is convenient to redefine the zero of energy, so that only the effective kinetic energy,

$$\epsilon(p_\perp) \cong p_\perp^2/2m, \quad (7)$$

remains. To establish the connection with the Bogoliubov Hamiltonian, we identify the two-dimensional momentum p_\perp as the momentum p that appears in this Hamiltonian, and the above $\epsilon(p_\perp)$ as the $\epsilon(p)$ that appears in Eq. (1).

IV. THE BOGOLIUBOV DISPERSION RELATION FOR THE PHOTON FLUID

Now we know that in an ideal Bose gas at absolute zero temperature, there exists a Bose condensate consisting of a macroscopic number N_0 of particles occupying the zero-momentum state. This feature should survive in the case of the weakly-interacting Bose gas, since as the interaction vanishes, one should recover the Bose condensate state. Hence following Bogoliubov, we shall assume here that even in the presence of interactions, N_0 will remain a macroscopic number in the photon fluid [11]. This macroscopic number will be determined by the intensity of the incident laser beam which excites the Fabry-Perot cavity system, and turns out to be a very large number compared to unity (see below). For the ground state wave function $\Psi_0(N_0)$ with N_0 particles in the Bose condensate in the $p = 0$ state, the zero-momentum operators a_0 and a_0^\dagger operating on the ground state obey the relations

$$\begin{aligned} a_0 |\Psi_0(N_0)\rangle &= \sqrt{N_0} |\Psi_0(N_0 - 1)\rangle \\ a_0^\dagger |\Psi_0(N_0)\rangle &= \sqrt{N_0 + 1} |\Psi_0(N_0 + 1)\rangle. \end{aligned} \quad (8)$$

Since $N_0 \gg 1$, we shall neglect the difference between the factors $\sqrt{N_0 + 1}$ and $\sqrt{N_0}$. Thus one can replace all occurrences of a_0 and a_0^\dagger by the c -number $\sqrt{N_0}$, so that to a good approximation $[a_0, a_0^\dagger] \approx 0$. However, the number of particles in the system is then no longer exactly conserved, as can be seen by examination of the term in the Hamiltonian

$$\sum_{\kappa} V(\kappa) a_{\kappa}^\dagger a_{-\kappa}^\dagger a_0 a_0 \approx N_0 \sum_{\kappa} V(\kappa) a_{\kappa}^\dagger a_{-\kappa}^\dagger, \quad (9)$$

which represents the creation of a pair of particles, i.e., photons, with transverse momenta κ and $-\kappa$ out of nothing.

However, whenever the system is open one, i.e., whenever it is connected to an external reservoir of particles which allows the total particle number inside the system (i.e., the cavity) to fluctuate around some constant average value, then the total number of particles need only be conserved on the average. Formally, one standard way to compensate for the lack of exact particle number conservation is to use the Lagrange multiplier method and

subtract a chemical potential term μN_{op} from the Hamiltonian (just as in statistical mechanics when one goes from the canonical ensemble to the grand canonical ensemble) [13]

$$H \rightarrow H' = H - \mu N_{op}, \quad (10)$$

where $N_{op} = \sum_p a_p^\dagger a_p$ is the total number operator, and μ represents the chemical potential, i.e., the average energy for adding a particle to the open system described by H . In the present context, we are considering the case of a Fabry-Perot cavity with low, but finite, transmittivity mirrors which allow photons to enter and leave the cavity, due to an input light beam coming in from the left and an output beam leaving from the right (see Fig. 3). This permits a realistic physical implementation of the external reservoir, since the Fabry-Perot cavity allows the total particle number inside the cavity to fluctuate due to particle exchange with the beams outside the cavity. However, the photons remain trapped inside the cavity long enough so that a condition of thermal equilibrium is achieved after multiple photon-photon interactions (i.e., after very many collisions, which is indeed the case for the experimental numbers to be discussed below). This leads to the formation of a photon fluid inside the cavity [14].

It will be useful to separate out the zero-momentum components of the interaction Hamiltonian, since it will turn out that there is a macroscopic occupation of the zero-momentum state due to Bose condensation. The prime on the sums \sum'_p , $\sum'_{p\kappa}$, and $\sum'_{\kappa pq}$ in the following equation denotes sums over momenta explicitly excluding the zero-momentum state, i.e., all the running indices $p, \kappa, q, p + \kappa, q - \kappa$ which are not explicitly set equal to zero, are nonzero:

$$\begin{aligned} H_{int} = & \frac{1}{2} V(0) a_0^\dagger a_0^\dagger a_0 a_0 + V(0) \sum'_p a_p^\dagger a_p a_0^\dagger a_0 + \\ & \sum'_p \left(V(p) a_p^\dagger a_0^\dagger a_p a_0 + \frac{1}{2} \left[V(p) a_p^\dagger a_{-p}^\dagger a_0 a_0 + V(p) a_0^\dagger a_0^\dagger a_p a_{-p} \right] \right) + \\ & \sum'_{p\kappa} V(\kappa) \left(a_{p+\kappa}^\dagger a_0^\dagger a_p a_\kappa + a_{p+\kappa}^\dagger a_{-\kappa}^\dagger a_p a_0 \right) + \frac{1}{2} \sum'_{\kappa pq} V(\kappa) \left(a_{p+\kappa}^\dagger a_{q-\kappa}^\dagger a_p a_q \right). \end{aligned} \quad (11)$$

Here we have also assumed that $V(p) = V(-p)$. By thus separating out the zero-momentum state from the sums in the Hamiltonian, and replacing all occurrences of a_0 and a_0^\dagger by $\sqrt{N_0}$, we find that the Hamiltonian H' in Eq. (10) decomposes into three parts

$$H' = H_0 + H_1 + H_2, \quad (12)$$

where, in decreasing powers of $\sqrt{N_0}$,

$$H_0 = \frac{1}{2} V(0) a_0^\dagger a_0^\dagger a_0 a_0 \approx \frac{1}{2} V(0) N_0^2, \quad (13)$$

$$H_1 \approx \sum'_p \epsilon'(p) a_p^\dagger a_p + \frac{1}{2} N_0 \sum'_p V(p) \left(a_{-p}^\dagger a_p^\dagger + a_{-p} a_p \right), \quad (14)$$

$$H_2 \approx \sqrt{N_0} \sum'_{p\kappa} V(\kappa) \left(a_{p+\kappa}^\dagger a_p a_\kappa + a_{p+\kappa}^\dagger a_{-\kappa}^\dagger a_p \right) + \frac{1}{2} \sum'_{\kappa pq} V(\kappa) \left(a_{p+\kappa}^\dagger a_{q-\kappa}^\dagger a_p a_q \right). \quad (15)$$

Here

$$\epsilon'(p) = \epsilon(p) + N_0 V(0) + N_0 V(p) - \mu \quad (16)$$

is a modified photon energy, where N_0 is given by

$$N_0 + \langle \Psi_0 | \sum'_p a_p^\dagger a_p | \Psi_0 \rangle = N \quad (17)$$

(the term $\langle \Psi_0 | \sum'_p a_p^\dagger a_p | \Psi_0 \rangle$ represents the number of photons in the “depletion,” i.e., those photons which have been scattered out of the condensate at $T = 0$ due to photon-photon collisions), and where μ is given by

$$\mu = \frac{\partial E_0}{\partial N}. \quad (18)$$

Here $E_0 = \langle \Psi_0 | H | \Psi_0 \rangle$ is the ground state energy of H . In the approximation that there is little depletion of the Bose condensate due to the interactions (i.e., $N \approx N_0 \gg 1$), the first term of Eq. (11) (i.e., H_0 in Eq. (13)) dominates, so that

$$E_0 \approx \frac{1}{2} N_0^2 V(0) \approx \frac{1}{2} N^2 V(0), \quad (19)$$

and therefore that

$$\mu \approx NV(0) \approx N_0 V(0). \quad (20)$$

This implies that the effective chemical potential of a photon, i.e., the energy for adding a photon to the photon fluid, is given by the number of photons in the Bose condensate times the repulsive pairwise interaction energy between photons with zero relative momentum. It should be remarked that the fact that the chemical potential is nonvanishing here makes the thermodynamics of this two-dimensional photon system quite different from the usual three-dimensional, Planck blackbody photon system. It should also be remarked that the conventional wisdom which tells us that Bose-Einstein condensation and superfluidity are impossible in 2D bosonic systems, does not apply here. To the contrary, we believe that superfluidity of the topological, 2D Kosterlitz-Thouless kind (with algebraic decay of long range order) is possible for the photon fluid [15].

In the same approximation $N \approx N_0 \gg 1$, Eq. (16) becomes, upon use of the fact that $\mu \approx N_0 V(0)$

$$\epsilon'(p) \approx \epsilon(p) + N_0 V(p). \quad (21)$$

This is the single-particle photon energy in the Hartree approximation. Here it is again assumed that $|H_1| \gg |H_2|$, i.e., that the interactions between the bosons are sufficiently weak, so as not to deplete the Bose condensate significantly. In the case of the weakly-interacting photon gas inside the Fabry-Perot resonator, since the interactions between the photons are indeed weak, this assumption is a good one.

Following Bogoliubov, we now introduce the following canonical transformation in order to diagonalize the quadratic-form Hamiltonian H_1 in Eq. (14):

$$\begin{aligned} \alpha_\kappa &= u_\kappa a_\kappa + v_\kappa a_{-\kappa}^\dagger \\ \alpha_\kappa^\dagger &= u_\kappa a_\kappa^\dagger + v_\kappa a_{-\kappa} \end{aligned} \quad (22)$$

Here u_κ and v_κ are two real c -numbers which must satisfy the condition

$$u_\kappa^2 - v_\kappa^2 = 1, \quad (23)$$

in order to insure that the Bose commutation relations are preserved for the new creation and annihilation operators for certain new quasi-particles, α_κ^\dagger and α_κ , i.e., that

$$[\alpha_\kappa, \alpha_{\kappa'}^\dagger] = \delta_{\kappa, \kappa'} \text{ and } [\alpha_\kappa, \alpha_{\kappa'}] = [\alpha_\kappa^\dagger, \alpha_{\kappa'}^\dagger] = 0. \quad (24)$$

We seek a diagonal form of H_1 given by

$$H_1 = \sum_\kappa' \left[\tilde{\omega}(\kappa) \left(\alpha_\kappa^\dagger \alpha_\kappa + \frac{1}{2} \right) + \text{constant} \right], \quad (25)$$

where $\tilde{\omega}(\kappa)$ represents the energy of a quasi-particle of momentum κ . Substituting the new creation and annihilation operators α_κ^\dagger and α_κ given by Eq. (22) into Eq. (25), and comparing with the original form of the Hamiltonian H_1 in Eq. (14), we arrive at the following necessary conditions for diagonalization:

$$\tilde{\omega}(\kappa) u_\kappa v_\kappa = \frac{1}{2} N_0 V(\kappa) \quad (26)$$

$$u_\kappa^2 = \frac{1}{2} [1 + \epsilon'(\kappa)/\tilde{\omega}(\kappa)] \quad (27)$$

$$v_\kappa^2 = \frac{1}{2} [-1 + \epsilon'(\kappa)/\tilde{\omega}(\kappa)]. \quad (28)$$

Squaring Eq. (26) and substituting from Eqs. (27) and (28), we obtain

$$\tilde{\omega}(\kappa)^2 = \epsilon'(\kappa)^2 - N_0^2 V(\kappa)^2 \approx \epsilon(\kappa)^2 + 2\epsilon(\kappa)N_0 V(\kappa), \quad (29)$$

where in the last step we have used Eq. (21).

Thus the final result is that the Hamiltonian H_1 in Eq. (25) describes a collection of *noninteracting* simple harmonic oscillators, i.e., quasi-particles, or elementary excitations of the photon fluid from its ground state. The energy-momentum relation of these quasi-particles is obtained from Eq. (29) upon substitution of $\epsilon(\kappa) = \kappa^2/2m$ from Eq. (7)

$$\tilde{\omega}(\kappa) \approx \left[\frac{\kappa^2 N_0 V(\kappa)}{m} + \frac{\kappa^4}{4m^2} \right]^{1/2}, \quad (30)$$

which we shall call the “Bogoliubov dispersion relation.” This dispersion relation is plotted in Fig. 2, in the special case that $V(\kappa) \approx V(0) = \text{constant}$. (Note that Landau’s roton minimum can also be incorporated into this theory by a suitable choice of the functional form of $V(\kappa)$.)

For small values of κ this dispersion relation is *linear* in κ , indicating that the nature of the elementary excitations here is that of *phonons*, which in the classical limit of large phonon number leads to sound waves propagating inside the photon fluid at the sound speed

$$v_s = \lim_{\kappa \rightarrow 0} \frac{\tilde{\omega}(\kappa)}{\kappa} = \left(\frac{N_0 V(0)}{m} \right)^{1/2} = \left(\frac{\mu}{m} \right)^{1/2}. \quad (31)$$

At a transition momentum κ_c given by

$$\kappa_c = 2(mN_0 V(\kappa_c))^{1/2} \quad (32)$$

(i.e., when the two terms of Eq. (30) are equal), the linear relation between energy and momentum turns into a quadratic one, indicating that the quasi-particles at large momenta behave essentially like nonrelativistic free particles with an energy of $\kappa^2/2m$. The reciprocal of κ_c defines a characteristic length scale

$$\lambda_c \equiv 2\pi\hbar/\kappa_c = \pi\hbar/mv_s, \quad (33)$$

which characterizes the distance scale over which collective effects arising from the pairwise interaction between the photons become important.

Thus in the above analysis, we have shown that all the approximations involved in the Bogoliubov theory should be valid ones for the case of the 2D photon fluid inside a nonlinear Fabry-Perot cavity. Hence the Bogoliubov dispersion relation should indeed apply to this fluid; in particular, there should exist sound wave modes of propagation in the photon fluid. As additional evidence for the existence of these modes, we have recently found that the same Bogoliubov dispersion relation emerges from a classical nonlinear optical analysis of this problem [6], which we shall not reproduce here. The velocity of sound found by the macroscopic, classical nonlinear optical analysis is identical to the one found in Eq. (31) for the velocity of phonons in the photon fluid in the above microscopic analysis, provided that one identifies the energy density of the light inside the cavity with the number of photons in the Bose condensate as follows:

$$\mathcal{E}_0^2 = 8\pi N_0 \hbar \omega / V_{cav}, \quad (34)$$

where V_{cav} , the cavity volume, is also the quantization volume for the electromagnetic field, and provided that one makes use of the known proportionality between the Kerr coefficient n_2 and the photon-photon interaction potential $V(0)$ [16,17]

$$V(0) = 8\pi(\hbar\omega)^2 n_2 / V_{cav}. \quad (35)$$

In fact, the entire dispersion relation found by the classical, macroscopic analysis for sound waves associated with fluctuations in the light intensity inside a resonator filled with a self-defocusing Kerr medium, turns out to be formally identical to the above Bogoliubov dispersion relation obtained quantum mechanically for the elementary excitations of the photon fluid, in the approximation $V(\kappa) \approx V(0) = \text{constant}$. This is a valid approximation, since the pairwise interaction potential between two photons is given by a transverse 2D pairwise spatial Dirac delta function, whose strength is proportional to n_2 , provided that the photons propagate paraxially, and the nonlinearity is fast. It should be kept in mind that the phenomena of self-focusing and self-defocusing in nonlinear optics can be viewed as arising from *pairwise interactions* between photons when the light propagation is paraxial and the Kerr nonlinearity is fast [16,17]. Since in a quantum description the light inside the resonator is composed of photons, and since these

photons as the constituent particles are weakly interacting repulsively with each other through the self-defocusing Kerr nonlinearity to form a photon fluid, this formal identification between the microscopic and macroscopic results for the Bogoliubov relation is a natural one [18].

One possible experiment to see these sound waves is sketched in Fig. 4. The sound wave mode is most simply observed by applying two incident optical fields to the nonlinear cavity: a broad plane wave resonant with the cavity to form the nonlinear background fluid on top of which the sound waves can propagate, and a weaker amplitude-modulated beam which is modulated at the sound frequency in the radio range by an electro-optic modulator, and injected by means of an optical fiber tip at a single point on the entrance face of the Fabry-Perot. The resulting weak time-varying perturbations in the background light induce transversely propagating density waves in the photon fluid, which propagate away from the point of injection like ripples on a pond. This sound wave can be phase-sensitively detected by another fiber tip placed at the exit face of the Fabry-Perot some transverse distance away from the injection point, and its sound wavelength can be measured by scanning this fiber tip transversely across the exit face.

The experiment could employ a cavity length L of 2 cm and mirrors with reflectivities of $R = 0.997$ for a cavity finesse $\mathcal{F} = 1050$. The optical nonlinearity could be provided by rubidium vapor at 80° C, corresponding to a number density of 10^{12} rubidium atoms per cubic centimeter. Incident on the cavity could be a circularly-polarized CW laser beam, detuned by around 600 MHz to the red side of a closed two-level transition, for example, the $|F = 2, m_F = +2\rangle \rightarrow |F' = 3, m_{F'} = +3\rangle$ transition of the ^{87}Rb D_2 line. Thus the Kerr nonlinear coefficient could be that of a pure two-level atomic system virtually excited well off resonance (i.e., with a detuning much larger than the absorption linewidth), which was calculated by Grischkowsky [19]:

$$n_2 = \pi N_{atom} \mu^4 / \hbar^3 \Delta^3 \approx 6 \times 10^{-6} \text{ cm}^3/\text{erg} \approx 5 \times 10^{-8} \text{ cm}^2/\text{Watt}, \quad (36)$$

where N_{atom} is the atomic number density of the atomic vapor, μ is the matrix element of the two-level atomic system, and Δ is the detuning of the laser frequency from the atomic resonance frequency. Thus the $\Delta \approx 600$ MHz detuning of the laser from the atomic resonance used in the above example would be considerably larger than the Doppler width of 340 MHz of the rubidium vapor, and the residual absorption arising from the tails of the nearby resonance line would give rise to a loss which would be less than or comparable to the loss arising from the mirror transmissions. This extra absorption loss would contribute to a slightly larger effective cavity loss coefficient, but would not otherwise alter the qualitative behavior of the Bogoliubov dispersion relation. The conditions of validity for the microscopic Bogoliubov theory should be well satisfied by these experimental parameters. An intracavity intensity of 40 W/cm^2 would result in $\Delta n = |n_2| \mathcal{E}_0^2 \approx 2 \times 10^{-6}$, for a sound speed $v_s \approx 4 \times 10^7 \text{ cm/s}$. For this intensity, $N_0 \approx 8 \times 10^{11}$, so that the condition for the validity of the Bogoliubov theory $N_0 \gg 1$ should be well satisfied. The cavity ring-down time $\tau_{cav} = 2\mathcal{F}L/c \approx 0.14 \mu\text{s}$ would be much longer than the mean photon-photon collision time $\tau_{coll} = (12\omega n_2 |\mathcal{E}_0|^2)^{-1} \approx 17 \text{ ps}$, so that a photon fluid should indeed form inside the cavity, since there would be approximately 8000 photon-photon collisions within a cavity ring-down time, so that the assumption of thermal equilibrium should be a valid one.

It should be noted that the above Bogoliubov theory is not limited to the above two-level atomic Kerr nonlinearity, which was chosen only for the purposes of illustration. One could replace this two-level nonlinearity with other recent, more promising kinds of nonlinearities, such as that in a four-level system, where absorption could be eliminated by the use of quantum interference while the Kerr nonlinearity could be simultaneously enhanced [20], or such as that due to photon exchange, where the nonlinearity is proportional to N_{atom}^2 rather than to N_{atom} [21].

V. DISCUSSION

We suggest here that the Bogoliubov form of dispersion relation, Eq. (30), implies that the photon fluid formed by the repulsive photon-photon interactions in the nonlinear cavity is actually a photon *superfluid*. This means that a superfluid state of light might actually exist. Although the exact definition of superfluidity is presently still under discussion, especially in light of the question whether the recently discovered atomic Bose-Einstein condensates are superfluids or not [4], one indication of the existence of a photon superfluid would be that there exists a critical transition from a dissipationless state of superflow, i.e., a laminar flow of the photon fluid below a certain critical velocity past an obstacle, into a turbulent state of flow, accompanied by energy dissipation associated with the shedding of a von-Karman street of *quantized* vortices past this obstacle, above this critical velocity. (It is the generation of *quantized* vortices above this critical velocity which distinguishes the onset of *superfluid* turbulence from the onset of *normal* hydrodynamic turbulence.)

The physical meaning of the Bogoliubov dispersion relation is that the lowest energy excitations of the system consist of quantized sound waves or phonon excitations in a superfluid, whose maximum critical velocity is then given by the sound wave velocity. By inspection of this dispersion relation, a single quantum of any elementary

excitation cannot exist with a velocity below that of the sound wave. Hence no excitation of the superfluid is possible at all for any object moving with a velocity slower than that of the sound wave velocity, according to an argument by Landau [22]. Hence the flow of the superfluid must be dissipationless below this critical velocity. Above a certain critical velocity, dissipation due to vortex shedding is expected from computer simulations based on the Gross-Pitaevskii (or Ginzburg-Landau or nonlinear Schrödinger) equation, which should give an accurate description of this system at the macroscopic level [23].

We propose a follow-up experiment to demonstrate that the sound wave velocity, typically a few thousandths of the vacuum speed of light, is indeed a maximum critical velocity of a fluid, i.e., that this photon fluid exhibits persistent currents in accordance with the Landau argument based on the Bogoliubov dispersion relation. Suppose we shine light at some nonvanishing incidence angle on a Fabry-Perot resonator (i.e., exciting it on some off-axis mode). This light produces a uniform flow field of the photon fluid, which flows inside the resonator in some transverse direction and at a speed determined by the incidence angle. A cylindrical obstacle placed inside the resonator will induce a laminar flow of the superfluid around the cylinder, as long as the flow velocity remains below a certain critical velocity. However, above this critical velocity a turbulent flow will be induced, with the formation of a von-Karman vortex street associated with quantized vortices shed from the boundary of the cylinder [23]. The typical vortex core size is given by the light wavelength divided by the square root of the nonlinear index change. Typically the vortex core size should be around a few hundred microns, so that this nonlinear optical phenomenon should be readily observable.

A possible application is suggested by an analogy with the Meissner effect in superconductors, or the Hess-Fairbank effect in superfluid helium: Vortices in an incident light beam would be expelled from the interior of the photon superfluid. This would lead to a useful beam-cleanup effect, in which speckles in a dirty incident laser beam would be expelled upon transmission through the nonlinear Fabry-Perot resonator, so that a clean, speckle-free beam emerges.

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equilibrium. The photon fluid is in thermal equilibrium with the external light beams (see Fig. 3) in the same sense that a superfluid helium thin film on a cold substrate is in thermal equilibrium with the helium atoms incident on it from the vapor, and also with the helium atoms evaporating from it into the vapor.

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FIGURES

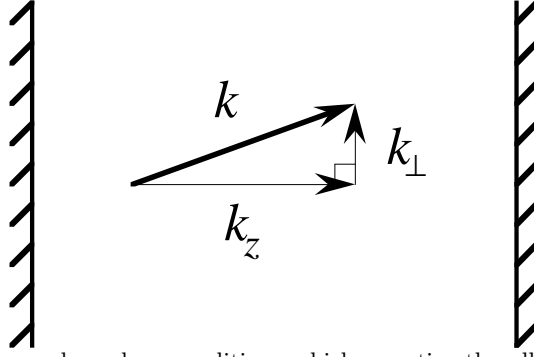


FIG. 1. A planar Fabry-Perot imposes boundary conditions which quantize the allowed values of k_z , where z is the axis normal to the mirrors, in units of π/L , where L is the separation of the mirrors. For a plane-wave mode which propagates at a small angle with respect to the z axis, there arises an effective nonrelativistic energy-momentum relation for an noninteracting, trapped 2D photon, whose effective mass is $m \approx \hbar\omega/c^2$ (see text).

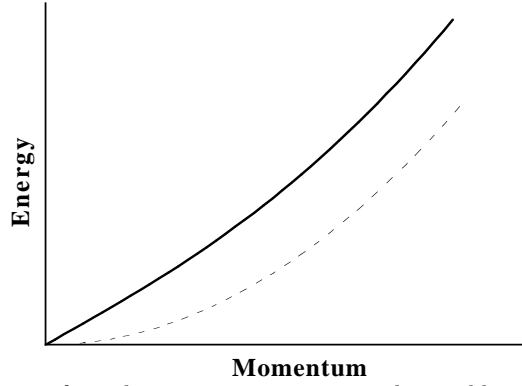


FIG. 2. The energy versus momentum of an elementary excitation in the weakly-interacting Bose gas, here in the present case, the photon fluid. The solid line represents the Bogoliubov dispersion relation given by Eq. (30), for the special case that $V(\kappa) \approx V(0) = \text{constant}$, and the dashed line represents a quadratic dispersion relation for a noninteracting, diffracting photon inside the Fabry-Perot resonator.

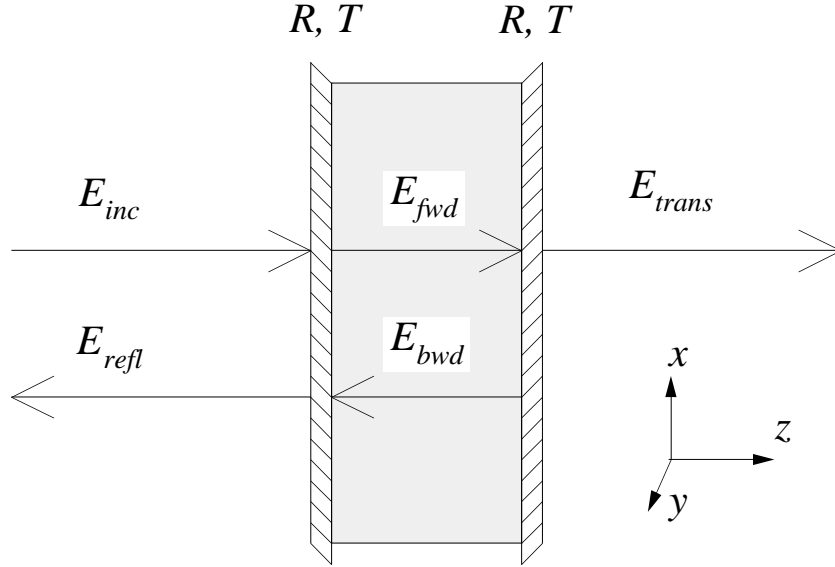


FIG. 3. Fields and coordinate system in the Fabry-Perot cavity. The applied field E_{inc} arises from a laser beam incident from the left. An atomic vapor excited to the red side of resonance by the incident light fills the space (the gray area) between the two mirrors. The presence of these atoms leads to a self-defocussing Kerr nonlinearity (corresponding to repulsive photon-photon interactions) inside the cavity.

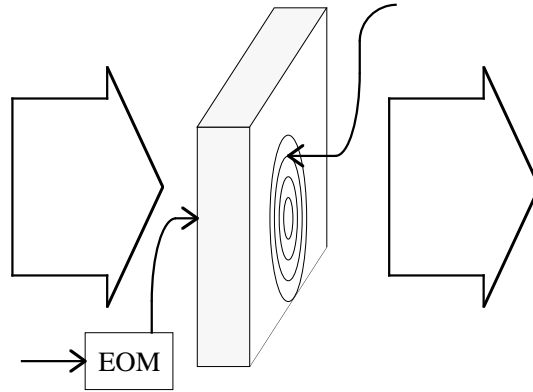


FIG. 4. Schematic of an experiment to observe the sound waves in a photon fluid which fills a nonlinear Fabry-Perot resonator. The nonlinear medium (denoted by the gray area) consists of atoms excited by a broad laser beam (denoted by the broad incoming arrow) well detuned to the red side of resonance. The goal is to verify the Bogoliubov dispersion relation, Eq. (30). An electro-optic modulator (EOM) modulates the intensity of light at a radio frequency, which is then injected by means of an optical fiber tip at a single point on the entrance face of the Fabry-Perot resonator. The wavelength of the resulting sound waves in the photon fluid can be measured by scanning in the transverse direction the tip of another optical fiber across the output face of the Fabry-Perot. A 2π phase shift of the modulated pick-up signal relative to that of the EOM modulation signal corresponds to a transverse displacement of the tip by a sound wavelength.